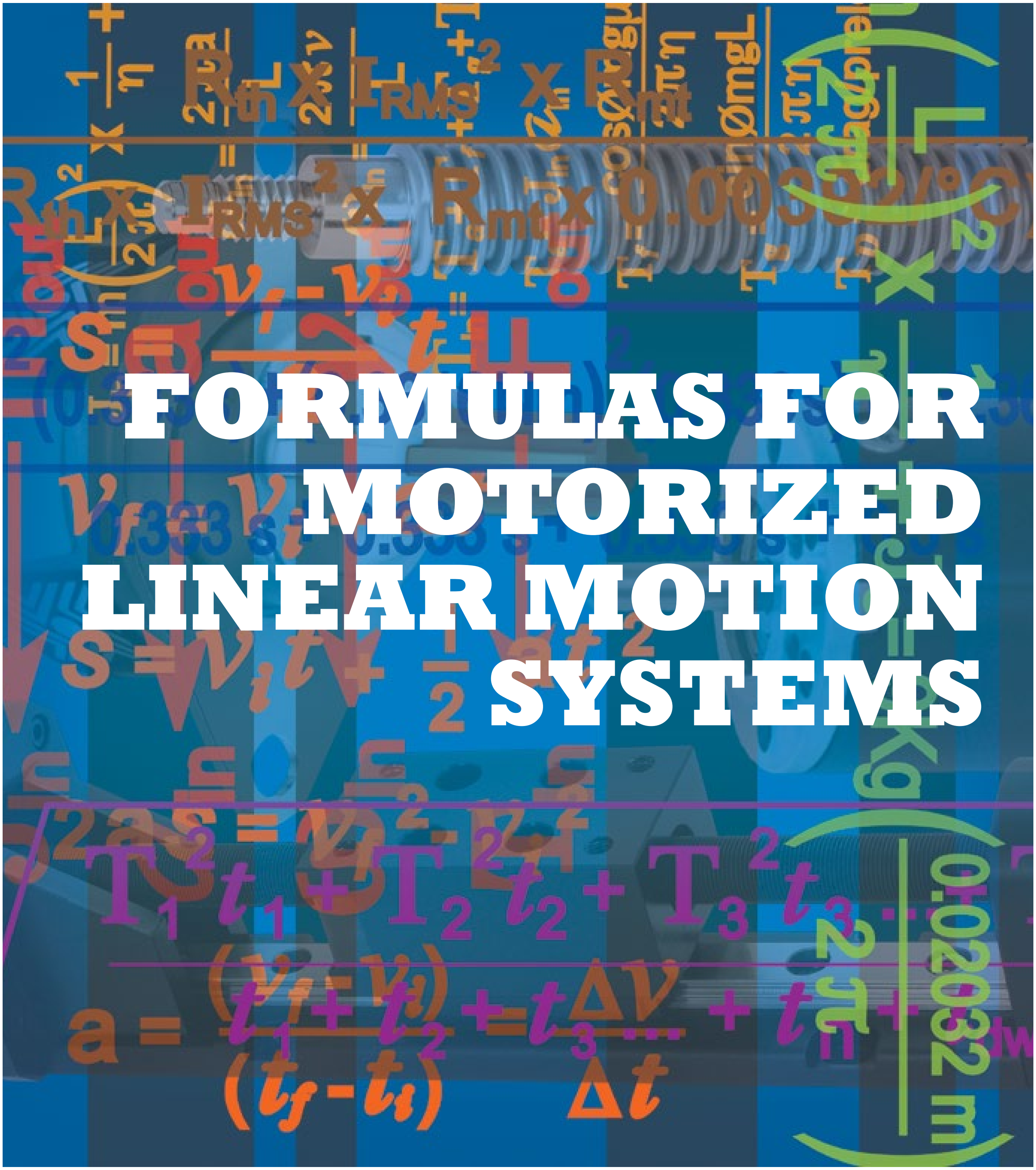


FORMULAS FOR MOTORIZED LINEAR MOTION SYSTEMS

Haydon kerk Motion Solutions® PITTMAN® | AMETEK® ADVANCED MOTION SOLUTIONS

Haydon Kerk Motion Solutions : **203 756 7441**
Pittman Motors : **267 933 2105**

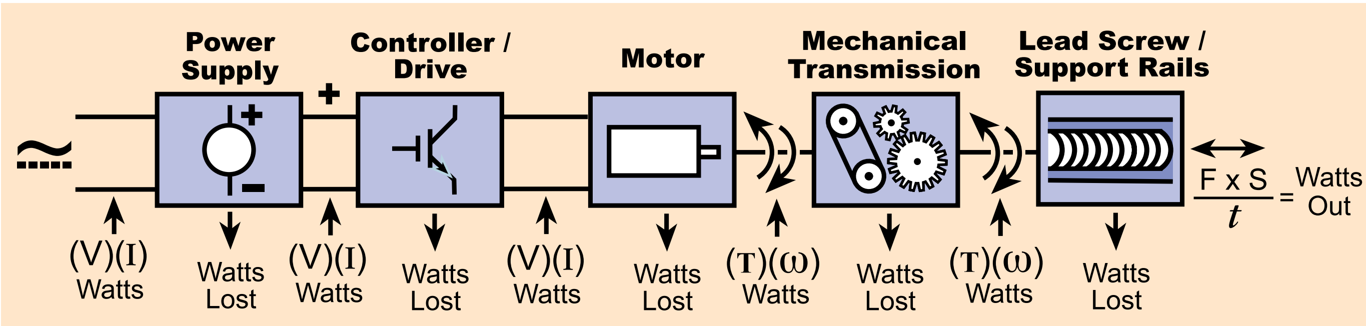
www.HaydonKerkPittman.com

SYMBOLS AND UNITS

Symbol	Description	Units
a	linear acceleration	m/s^2
α	angular acceleration	rad/s^2
α_{in}	angular acceleration	rad/s^2
α_{out}	angular acceleration	rad/s^2
α	temperature coefficient	/°C
D_V	viscous damping factor	Nm s/rad
F	force	N
F_a	linear force required during acceleration ($F_J + F_f + F_g$)	N
F_f	linear force required to overcome friction	N
F_g	linear force required to overcome gravity	N
F_J	linear force required to overcome load inertia	N
g	gravitational constant (9.8 m/s^2)	m/s^2
I	current	A
I_{LR}	locked rotor current	A
I_o	no-load current	A
I_{PK}	peak current	A
I_{RMS}	RMS current	A
J	inertia	$Kg\text{-}m^2$
J_B	inertia of belt	$Kg\text{-}m^2$
J_{GB}	inertia of the gear box	$Kg\text{-}m^2$
J_{in}	reflected inertia at system input	$Kg\text{-}m^2$
J_{out}	reflected inertia at system output	$Kg\text{-}m^2$
J_{P1}	inertia of pully 1	$Kg\text{-}m^2$
J_{P2}	inertia of pully 2	$Kg\text{-}m^2$
J_S	lead screw inertia	$Kg\text{-}m^2$
K_E	voltage constant	V/rad/s
$K_{(f)}$	K_E or K_T (final"hot")	Nm/A or V/(rad/s)
$K_{(i)}$	K_E or K_T (initial "cold")	Nm/A or V/(rad/s)
K_m	motor constant	$Nm/\sqrt{W}$
K_T	torque constant	Nm/A
L	screw lead	m/rev
m	mass	Kg
N	gear ratio	n/a
η	efficiency	n/a
n	speed	RPM
n_o	no load speed	RPM
n_{PK}	peak speed	RPM
θ	load orientation (horizontal = 0°, vertical = 90°)	degrees
Θ_a	ambient temperature	°C
Θ_f	motor temperature (final"hot")	°C
Θ_i	motor temperature (initial "cold")	°C
Θ_m	motor temperature	°C
Θ_r	motor temperature rise	°C
Θ_{rated}	rated motor temperature	°C

Symbol	Description	Units
P	power	W
P_{avg}	average power	W
P_{CV}	power required at constant velocity	W
P_{in}	power required at system input	W
P_{loss}	dissipated power	W
P_{max}	maximum power	W
$P_{max(f)}$	maximum power (final"hot")	W
$P_{max(i)}$	maximum power (initial "cold")	W
P_{out}	output power	W
P_{PK}	peak power	W
R_m	motor regulation	RPM/Nm
R_{mt}	motor terminal resistance	Ω
$R_{mt(f)}$	motor terminal resistance (final"hot")	Ω
$R_{mt(i)}$	motor terminal resistance (initial "cold")	Ω
R_{th}	thermal resistance	°C/W
r	radius	m
S	linear distance	m
t	time	s
T	torque	Nm
T_a	torque required to overcome load inertia	Nm
$T_{a\,(motor)}$	torque required at motor shaft during acceleration	Nm
T_C	continuous rated motor torque	Nm
T_{CF}	coulomb friction torque	Nm
T_d	reverse torque required for deceleration	Nm
T_D	drag / preload torque	Nm
T_f	torque required to overcome friction	Nm
T_g	torque required to overcome gravity	Nm
T_{in}	torque required at system input	Nm
T_{LR}	locked rotor torque	Nm
T_{out}	torque required at system output	Nm
T_{RMS}	RMS torque required over the total duty cycle	Nm
$T_{RMS\,(motor)}$	RMS torque required at the motor shaft	Nm
μ	coefficient of friction	n/a
V_T	motor terminal voltage	V
V_{bus}	DC drive bus voltage	V
v	linear velocity	m/s
v_f	final linear velocity	m/s
v_i	initial linear velocity	m/s
v_{PK}	peak linear velocity	m/s
W	energy	J
ω	angular velocity	rad/s
ω_{in}	angular velocity at system input	rad/s
ω_{out}	angular velocity at system output	rad/s
ω_o	no load angular velocity	rad/s
ω_{PK}	peak angular velocity	rad/s

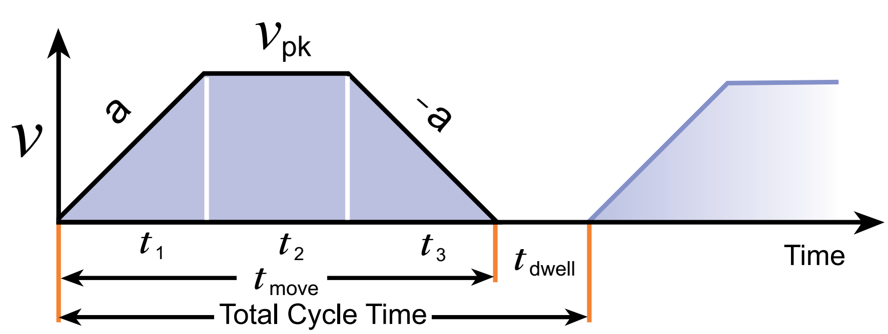
Any motion system should be broken down into its individual components.



All analysis begins with the linear motion profile of the output and the force required to move the load.

Motion Profiles

1/3 1/3 1/3 Trapazoidal Motion Profile



Optimized for minimum power

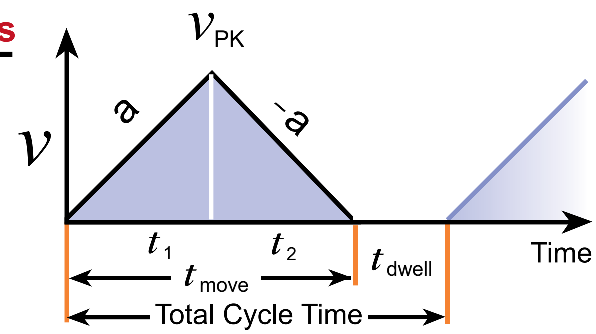
$$t_1 = t_2 = t_3$$

$$v_{PK} = \frac{3S}{2t}$$

S is the **total** move distance
t is the **total** move time
Area under profile curve represents distance moved

MOTION SYSTEM FORMULAS

Triangular Motion Profiles



Optimized for minimum acceleration slope

$$t_1 = t_2$$

$$v_{PK} = \frac{2S}{t}$$

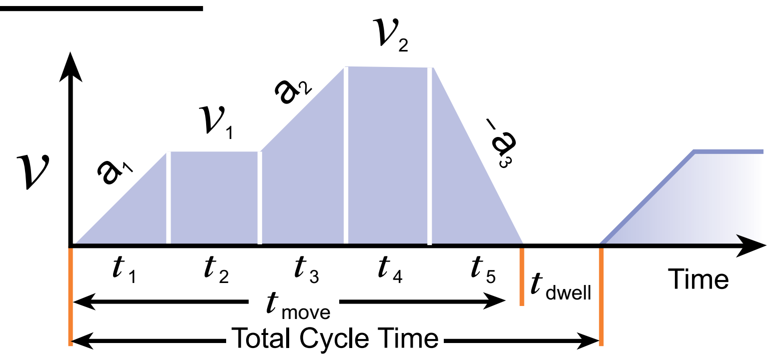
S is the **total** move distance

t is the **total** move time

Area under profile curve represents distance moved

Move Profile	Peak Velocity (V_{PK})	Acceleration (a)
1/3 1/3 1/3	↓	↑
Triangular	↑	↓

Complex Motion Profile



Linear Motion Formulas

(1) $S = \frac{v_f - v_i}{2} t$

(2) $v_f = v_i + at$

(3) $S = v_i t + \frac{1}{2} at^2$

(4) $2aS = v_f^2 - v_i^2$

(5) $a = \frac{(v_f - v_i)}{(t_f - t_i)} = \frac{\Delta v}{\Delta t}$

Linear to Rotary Formulas through a Lead Screw

$$\alpha = \frac{2\pi a}{L} = \text{rad/s}^2$$

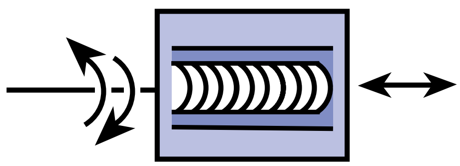
$$\omega = \frac{2\pi v}{L} = \text{rad/s}$$

$$n = \frac{60 v}{L} = \text{RPM}$$

3 known quantities are needed to solve for the other 2.
Each segment calculated individually.

Inertia, Acceleration, Velocity, and Required Torque

Lead Screw System



$J_{in} \leftarrow m_{out}$

$\alpha_{in} \leftarrow a_{out}$

$\omega_{in} \leftarrow v_{out}$

$T_{in} \leftarrow F_{out}$

$$J_{in} = m\left(\frac{L}{2\pi}\right)^2 \times \frac{1}{\eta} + J_S \quad \text{Kg-m}^2$$

$$\alpha_{in} = \frac{2\pi a}{L} \quad \text{rad/sec}^2$$

$$\omega_{in} = \frac{2\pi v}{L} \quad \text{rad/sec}$$

$$T_{in} = T_a + T_f + T_g + T_D \quad \text{N-m}$$

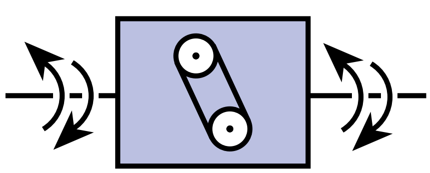
$$T_a = J_{in}\alpha_{in} \quad \text{N-m}$$

$$T_f = \frac{\cos\varnothing m g \mu L}{2\pi\eta} \quad \text{N-m}$$

$$T_g = \frac{\sin\varnothing m g L}{2\pi\eta} \quad \text{N-m}$$

T_D = drag/preload N-m
refer to manufacturer's data

Belt and Pulley System



$J_{in} \leftarrow J_{out}$

$\alpha_{in} \leftarrow \alpha_{out}$

$\omega_{in} \leftarrow \omega_{out}$

$T_{in} \leftarrow T_{out}$

$$J_{in} = \left(\frac{J_{out}}{N^2}\right) \times \frac{1}{\eta} + J_{P1} + J_{P2} + J_B \quad \text{Kg-m}^2$$

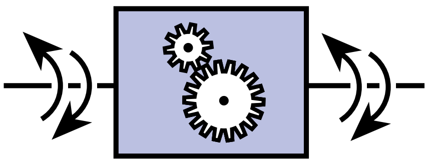
$$\alpha_{in} = (\alpha_{out})(N) \quad \text{rad/sec}^2$$

$$\omega_{in} = (\omega_{out})(N) \quad \text{rad/sec}$$

$$T_{in} = \frac{T_{out}}{N} \times \frac{1}{\eta} \quad \text{N-m}$$

For a 1st approximation analysis, J_{P1}, J_{P2}, and J_B can be disregarded. For higher precision, pulley and belt inertias should be included, as well as the reflected inertia of these components.

Gearbox System



$J_{in} \leftarrow J_{out}$

$\alpha_{in} \leftarrow \alpha_{out}$

$\omega_{in} \leftarrow \omega_{out}$

$T_{in} \leftarrow T_{out}$

$$J_{in} = \left(\frac{J_{out}}{N^2}\right) \times \frac{1}{\eta} + J_{GB} \qquad \text{Kg-m}^2$$

$$\alpha_{in} = (\alpha_{out})(N) \qquad \text{rad/sec}^2$$

$$\omega_{in} = (\omega_{out})(N) \qquad \text{rad/sec}$$

$$T_{in} = \frac{T_{out}}{N} \times \frac{1}{\eta} + T_D \qquad \text{N-m}$$

Drag torque can be significant (T_D) depending on the viscosity of the lubricant. For a first approximation analysis, this can be left out.

Gearbox inertia may be found in manufacturer's data sheets, or calculated using gear dimensions and materials / mass.

Mechanical Power and RMS Torque

$$\text{Linear Power} = \frac{FS}{t} = \text{watts}$$

$$\text{Rotary Power} = T\omega = \text{watts}$$

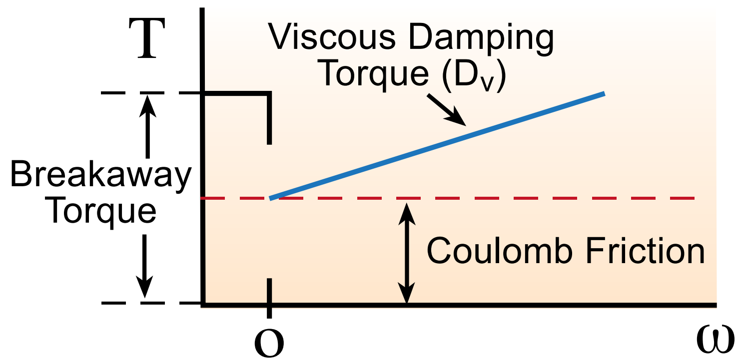
$$T_{RMS} = \sqrt{\frac{T_1^2 t_1 + T_2^2 t_2 + T_3^2 t_3 \ldots + T_n^2 t_n}{t_1 + t_2 + t_3 \ldots + t_n + t_{dwell}}}$$

DC Motor Formulas

Motor No Load Speed

$$n_o = 9.5493 \times \frac{V_T - (I_O \times R_{mt})}{K_E}$$

Assuming I_O is very small, it can be ignored for a quick approximation of motor no-load speed. If more accurate results are needed, I_O will also need to be adjuisted based on the motor Viscous Damping Factor (D_V). As the no-load speed increases with increased terminal voltage on a given motor, I_O will also increase based on the motor's D_V value found on the manufacturer's motor data sheet.



Motor Locked Rotor Current / Locked Rotor Torque

$$I_{LR} = \frac{V_T}{R_{mt}}$$

$$T_{LR} = I_{LR} \times K_T$$

$$= \frac{V_T}{R_{mt}} \times K_T$$

Motor Regulation

– Theoretical using motor constants

$$R_m = 9.5493 \times \frac{R_{mt}}{K_E \times K_T}$$

– Regulation calculated using test data

$$R_m = \frac{n_o}{T_{LR}}$$

Motor Power Relationships

– Watts lost due to winding resistance

$$P_{loss} = I^2 \times R_{mt}$$

– Output power at any point on the motor curve

$$P_{out} = \omega \times T$$

– Motor maximum output power

$$P_{max} = 0.25 \times \omega_o \times T_{LR}$$

– Motor maximum output power (Theoretical)

$$P_{max} = 0.25 \times \frac{V_T^{\,2}}{R_{mt}}$$

Temperature Effects on Motor Constants

– Change in terminal resistance

$$R_{mt(f)} = R_{mt(i)} \times [\, 1 + \alpha_{conductor} (\Theta_f - \Theta_i) \,]$$

$$\alpha_{conductor} = 0.0040/°C \text{ (copper)}$$

In the case of a mechanically commutated motor with graphite brushes, this analysis will result in a slight error because of the negative temperature coefficient of carbon. For estimation purposes, this can be ignored, but for a more rigorous analysis, the behavior of the brush material must be taken into account. This is not an issue when evaluating a brushless dc motor.

– Change in torque constant and voltage constant ($K_T = K_E$)

$$K_{(f)} = K_{(i)} \times [\, 1 + \alpha_{magnet} (\Theta_f - \Theta_i) \,]$$

Magnetic Material	α_{magnet} (/°C)	Θ_{max} (°C)
Ceramic	-0.0020/°C	300°C
SmCo	-0.0004/°C	300°C
AlNiCo	-0.0002/°C	540°C
NdFeB	-0.0012/°C	150°C

The magnetic material values above represent average figures for particular material classes and estimating performance. If exact values are needed, consult data sheets for a specific magnet grade.

Motor Brush Resistance

For PMDC motors (brush type) brush drop can be approximated as a constant resistance connected in series with the armature winding. This series resistance is included in the R_{mt} factor found on manufacturer's data sheet.

Brush Material	Resistance
Copper graphite	0.2 to 0.4 Ω
Silver graphite	0.2 to 0.4 Ω
Electrographite	0.8 to 1.0 Ω

DC Motor Thermal Resistance and Temperature Rise

– Thermal resistance

$$R_{th} = \frac{\text{Temperature Rise °C}}{\text{Machine Losses W}}$$

– Final motor temperature

$$\Theta_m = \Theta_r + \Theta_a$$

– Motor temperature rise*

$$\Theta_r = R_{th} \times I^2 \times R_{mt}$$

– OR –

$$\Theta_r = \frac{R_{th} \times I^2 \times R_{mt}}{1 - (R_{th} \times I^2 \times R_{mt} \times \alpha)}$$

– OR –

$$\Theta_r = R_{th} \times \left(\frac{T_C}{K_m}\right)^2$$

*Use caution when applying these formulas. Depending on the conditions used to determine motor specifications, only one formula should be used. Contact Appications Engineering for questions.

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*Use caution when applying these formulas. Depending on the conditions used to determine motor specifications, only one formula should be used. Contact Appications Engineering for questions.

Stepper Motor Formulas

– Motor step angle

$$\text{Step } Ø = \frac{180}{(\text{phases})(\text{rotor teeth})}$$

– SPS (steps/second) to RPM (revolutions/minute)

$$\text{RPM} = \frac{(\text{SPS}) \times (\text{Step } Ø)}{6}$$

– RPM (revolutions/minute) to SPS (steps/second)

$$\text{SPS} = \frac{(\text{RPM}) \times 6}{(\text{Step } Ø)}$$

– Travel per step

$$\text{m/step} = \frac{L}{\text{positions/rev}}$$

Maximum ***accuracy*** of a step motor system has nothing to do with maximum ***resolution*** of a step motor system. Maximum accuracy is always based on the accuracy of 1 full step, but maximum resolution is based on all system components including lead screw, encoder, stepper motor and step mode on the controller.

– Overdriving capability

Required "Time Off" using an **L/R drive** (constant voltage drive)

$$\text{Time Off} = (\text{Time On}) \frac{(\text{Applied Voltage})^2}{(\text{Rated Voltage})} - \text{Time On}$$

Required "Time Off" using a **chopper drive** (constant current drive)

$$\text{Time Off} = (\text{Time On}) \frac{(\text{Applied Current})^2}{(\text{Rated Current})} - \text{Time On}$$

It is not unusual for a customer to drive Haydon Kerk stepper motors beyond their rated power to obtain the most force in the smallest package size. Precautions must be taken to prevent the motor from exceeding its maximum temperature. The "on" time should not exceed 2 - 3 minutes. As general rule of thumb, driving a motor at 2.5 to 3x its rated voltage or current may result in wasted energy and erratic behavior due to saturation.

Special notes for can-stack motors:

- more easily saturated due to less active material (steel)
- more iron losses due to unlaminated steel

Lead Screw Performance Characteristics

Increase In	Affects	Impact
Screw Length	Critical speed	↓
	Compression load	↓
Screw Diameter	Critical speed	↑
	Inertia	↑
	Compression load	↑
	Stiffness	↑
	Spring Rate	↑
	Load capacity	↑
Lead	Drive torque	↑
	Angular velocity	↓
	Load cpacity	↑
	Positioning accuracy	↓

Increase In	Affects	Impact
End Mounting Rigidity	Critical speed	↑
	Compression load	↑
	System stiffness	↑
Load	Life	↓
Preload	Positioning accuracy	↑
	System stiffness	↑
	Drag torque	↑
Nut length	Load capacity	↑
	Stiffness	↑

Examples:
An INCREASE (↑) in screw length results in a DECREASE (↓) in critical speed.

An INCREASE in screw diameter results in an INCREASE in critical speed.

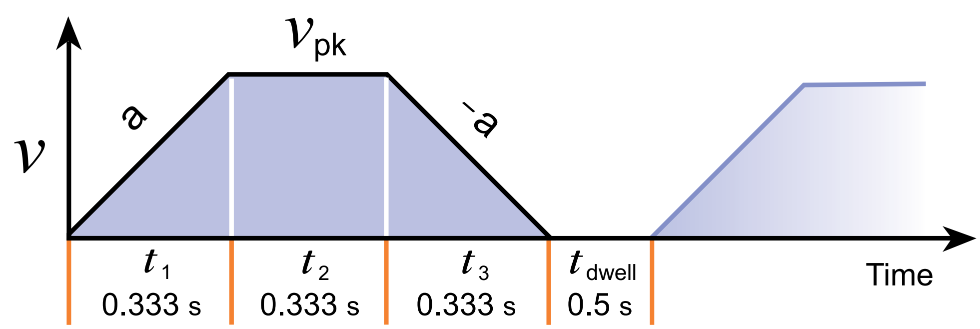
SI Unit Systems

Quantity	Name of Unit	Symbol	
Base Units			
Length	meter	m	
Mass	kilogram	Kg	
Time	second	s	
Electric current	ampere	A	
Thermodynamic temperature	kelvin	K	
Luminous intensity	candela	cd	
Amount of substance	mole	mol	
Derived Units			
area	square meter	m^2	
volume	cubic meter	m^3	
frequency	hertz	Hz	s^{-1}
mass density (density)	kilogram per cubic meter	Kg/m^3	
speed, velocity	meter per second	m/s	
angular velocity	radian per second	rad/s	
acceleration	meter per second squared	m/s^2	
angular acceleration	radian per second squared	rad/s^2	
force	newton	N	$Kg\text{-}m/s^2$
pressure (mechanical stress)	pascal	Pa	N/m^2
kinematic viscosity	square meter per second	m^2/s	
dynamic viscosity	newton-second per square meter	$N\text{-}s/m^2$	
work, energy, quantity of heat	joule	J	N-m
power	watt	W	J/s or N-m/s
entropy	joule per kelvin	J/K	
specific heat capacity	joule per kilogram kelvin	J/Kg-K)	
thermal conductivity	watt per meter kelvin	W/(m-K)	

Linear Motion Example

Data

Mass	9 Kg	Control	4 quadrant with encoder
Move distance	200 mm	Drive Power Supply:	32 VDC, 3.5 Arms, 5.0 A peak
Move time	1.0 s		
Dwell time	0.5 s		
Motion profile	1/3 1/3 1/3 Trapezoidal		
Screw speed limit	1000 RPM		
Load support	$\mu = 0.01$		
Orientation	Vertical		
Rotary to linear conversion	TFE lead screw 8 mm diameter 275 mm long		
Ambient temperature	30°C		



Linear Velocity

$$v_{PK} = \frac{3S}{2t} = \frac{(3)(0.2\text{m})}{(2)(1.0\text{ s})} = \frac{0.6\text{m}}{2\text{ s}} = \boxed{\frac{0.3\text{m}}{\text{s}}}$$

Linear Acceleration

$$a = \frac{v_f - v_i}{t} = \frac{0.3\text{m/s} - 0\text{ m/s}}{0.333\text{ s}} = \boxed{0.901\text{m/s}^2}$$

1st step is to thoroughly understand system constraints and motion profile requirements.

Minimum screw lead to maintain < 1000 RPM shaft speed

$$L_{min} = \frac{60 v_{PK}}{n} = \frac{(60)(0.3\text{m/s})}{1000 \text{ rev/min}} = 0.018\text{m/rev} = 18\text{mm/rev}$$

Refer to Kerk screw chart – closest lead in an 8mm screw diameter is

$$20.32\text{mm/rev} = \boxed{0.02032 \text{ m/rev}}$$

$\eta = 0.86$ free wheeling nut, TFE coated screw

$$J_S = 38.8 \times 10^{-7} \text{ Kg-m}^2$$

Other critical lead screw considerations

- Column loading
- Critical speed

Linear to Rotary Conversion – Velocity, Acceleration, Inertia, Torque

$v_{PK} \longrightarrow \omega_{PK}$ $m \longrightarrow J$

$a \longrightarrow \alpha$ $F \longrightarrow T$

Linear to Rotary Conversion: Velocity

$$\omega_{PK} = \frac{2\pi v_{PK}}{L} = \frac{(2\pi)(0.3\text{m/s})}{0.02032 \text{ m/rev}} = \boxed{92.76 \text{ rad/s}}$$

Linear to Rotary Conversion: Acceleration

$$\alpha = \frac{2\pi a}{L} = \frac{(2\pi)(0.901\text{m/s}^2)}{0.02032 \text{ m/rev}} = \boxed{278.6 \text{ rad/s}^2}$$

Linear to Rotary Conversion: Inertia

$$J_{in} = m\left(\frac{L}{2\pi}\right)^2 \mathbf{x} \frac{1}{\eta} + J_S = 9Kg\left(\frac{0.02032\ m}{2\pi}\right)^2 \mathbf{x} \frac{1}{0.86} + 38.8x10^{-7} Kg\text{-}m^2$$

$$= 10.95 \mathbf{\ x\ } 10^{-5} Kg\text{-}m^2 + 38.8x10^{-7} Kg\text{-}m^2$$

$$= \boxed{11.34 \text{ x } 10^{-5} Kg\text{-}m^2}$$

Linear to Rotary Conversion: Torque

$$T_{in} = T_a + T_f + T_g + T_D$$

$$T_a = J_{in}\alpha_{in} = (11.34 \text{ x } 10^{-5} Kg\text{-}m^2)(278.6\ rad/s^2)$$

$$= \boxed{0.0316\ Nm}$$

$$T_f = \frac{\cos Ømg\mu L}{2\pi\eta} = \frac{(\cos 90)(9\ Kg)(9.8 m/s^2)(0.01)(0.02032 m)}{2\pi(0.86)}$$

$$= \boxed{0\ Nm}$$

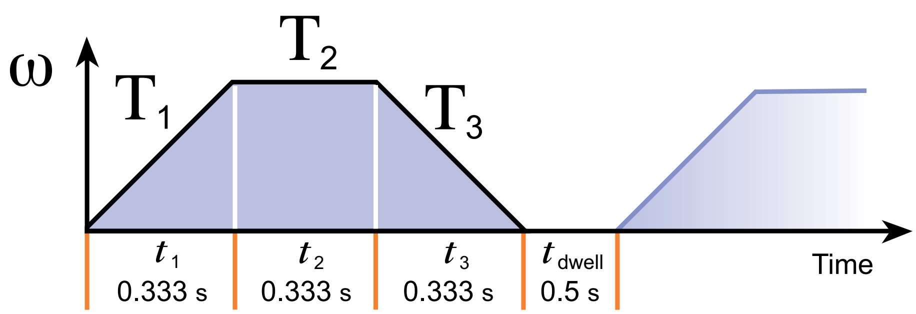
$$T_g = \frac{\sin ØmgL}{2\pi\eta} = \frac{(\sin 90)(9\ Kg)(9.8 m/s^2)(0.02032 m)}{2\pi(0.86)}$$

$$= \frac{1.792\ Nm}{5.404} = \boxed{0.3316\ Nm}$$

$$T_D = \boxed{0\ Nm}$$

Since a free-wheeling lead screw nut was used, there is no preload force. If an anti-backlash nut was used, T_D would be >0 due to preload force. Always reference manufacturer's data for more information.

Rotary Motion Profile



ω_{PK} = 92.76 rad/s α = 278.6 rad/s^2

$$T_1 = T_a + T_f + T_g + T_D$$

= (0.0316 Nm) + (0 Nm) + (0.3316 Nm) + (0 Nm)

= **0.3632 Nm**

$$T_2 = T_a + T_f + T_g + T_D$$

= (0 Nm) + (0 Nm) + (0.3316 Nm) + (0 Nm)

= **0.3316 Nm**

$$T_3 = T_a + T_f + T_g + T_D$$

= (0.0316 Nm) + (0 Nm) + (0.3316 Nm) + (0 Nm)

= **0.3632 Nm**

Lead Screw Input Requirements

ω_{PK} = 92.76 rad/s
a = 278.6 rad/s^2
T_1 = 0.3632 Nm
T_2 = 0.3316 Nm
T_3 = – 0.3632 Nm
t_1 = 0.333 s
t_2 = 0.333 s
t_3 = 0.333 s
t_{dwell} = 0.5 s
P_{PK} = 33.69 W
P_{CV} = 30.76 W

$$P_{PK} = (T_1)(\omega_{PK}) = (0.3632\text{ Nm})(92.76\text{ rad/s}) = \boxed{33.69\text{ W}}$$

$$P_{CV} = (T_2)(\omega_{PK}) = (0.3316\text{ Nm})(92.76\text{ rad/s}) = \boxed{30.76\text{ W}}$$

RMS Torque Requirement @ Lead Screw Input

$$T_{RMS} = \sqrt{\frac{T_1^2 t_1 + T_2^2 t_2 + T_3^2 t_3}{t_1 + t_2 + t_3 + t_{dwell}}}$$

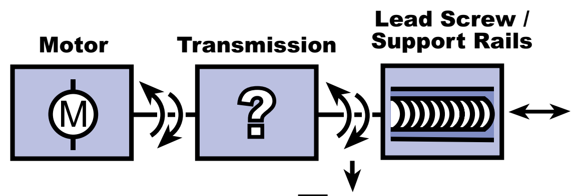
$$= \sqrt{\frac{(0.3632\text{Nm})^2(0.333\text{s})+(0.3316\text{Nm})^2(0.333\text{s})+(-0.3632\text{Nm})^2(0.333\text{s})}{0.333\text{ s} + 0.333\text{ s} + 0.333\text{ s} + 0.5\text{ s}}}$$

$$= \sqrt{\frac{0.0439 + 0.0366 + 0.0439}{1.5}} = \sqrt{0.0829}$$

$$T_{RMS} = \boxed{0.2879\text{ Nm}}$$

Load Parameters at the Lead Screw Shaft

ω_{PK} = 92.76 rad/s
a = 278.6 rad/s^2
T_1 = 0.3632 Nm
T_2 = 0.3316 Nm
T_3 = – 0.3632 Nm
T_{RMS} = 0.2879 Nm
P_{PK} = 33.69 W
P_{CV} = 30.76 W



T_{RMS} = 0.2879 Nm
P_{PK} = 33.69 W
ω_{PK} = 92.76 rad/s

Other Motor Considerations

– Type: stepper, brush, BLDC
– Input speed (high input speed will result in high noise levels)
– Total motor footprint
– Ambient temperature
– Environmental conditions
– Load-to-rotor inertia
– Encoder ready needed

Motor Selection

A direct-drive motor can be used, however, it would be large and expensive. For example, a DC057B-3 brush motor...

...would easily meet continuous torque and continuous speed, as well as a 1.8:1 load-to-rotor inertia. This motor, however, has significantly more output power than needed at 128 watts.

A smaller motor with a transmission would optimize the system. Look for a motor starting at a rated power around **40 watts**... Pittman DC040B-6

DC040B-6

Reference Voltage	24.0 V
Continuous Torque	0.0812 Nm
Rated Current	2.36 A
Rated Power	37 W
Torque Constant	0.042 Nm/A
Voltage Constant	0.042 V/rad/s
Terminal Resistance	1.85 Ω
Max. Winding Temperature	155°C
Rotor Inertia	8.47 x 10^{-6} Kg-m^2

Gearbox / Transmission Selection

When selecting a reduction ratio, keep in mind that the RMS required torque at the motor shaft needs to fall below the continuous rated output torque of the motor.

G30 A - Planetary Gearbox

Maximum output load 2.47 Nm

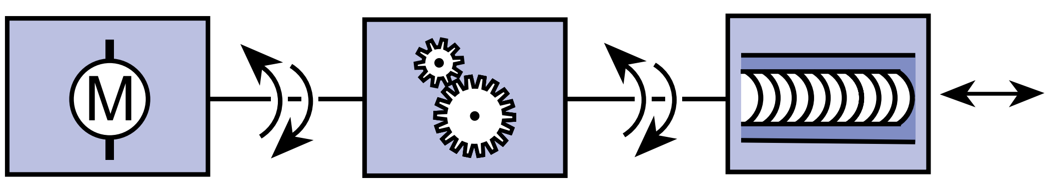
$T_{RMS(@\ Lead\ Screw)}$	Reduction	Efficiency	$T_{RMS(@\ Motor)}$
0.2879 Nm	4:1	0.90	0.0798 Nm
0.2879 Nm	**5:1**	**0.90**	**0.0640 Nm**
0.2879 Nm	6:1	0.90	0.0533 Nm

A 5:1 gearbox would allow about a 27% safety margin between what the system requires and the continuous torque output of the motor.

System Summary

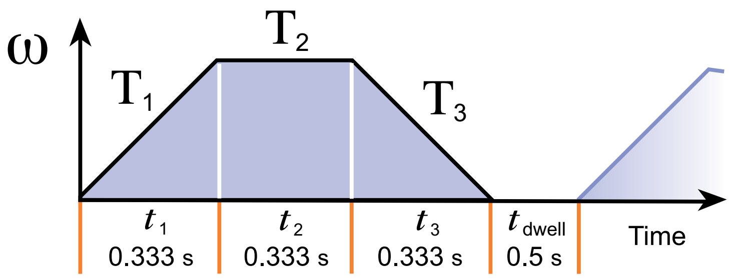
5:1

$\eta = 0.90$



$T_1 = 0.0807$ Nm	←	$T_1 = 0.3632$ Nm
$T_2 = 0.0737$ Nm	←	$T_2 = 0.3316$ Nm
$T_3 = -0.0807$ Nm	←	$T_3 = -0.3632$ Nm
$T_{RMS} = 0.0640$ Nm	←	$T_{RMS} = 0.2879$ Nm
$\omega_{PK} = 463.8$ rad/s	←	$\omega_{PK} = 92.76$ rad/s
$J = 5.04 \times 10^{-6}$ Kg-m^2	←	$J = 11.34 \times 10^{-5}$ Kg-m^2
$P_{PK} = 37.43$ W	←	$P_{PK} = 33.69$ W

Motion Profile at the Motor



ω_{PK} = 463.8 rad/s

n_{PK} = 4429 RPM

$T_1 = T_{PK}$ = 0.0807 Nm

Drive and Power Supply Requirements

Peak Current Required

$$I_{PK} = \frac{T_{PK}}{K_T} + I_O = \frac{0.0807\ \text{Nm}}{0.042\ \text{Nm/A}} + 0.180\ \text{A} = \boxed{2.10\ \text{A}}$$

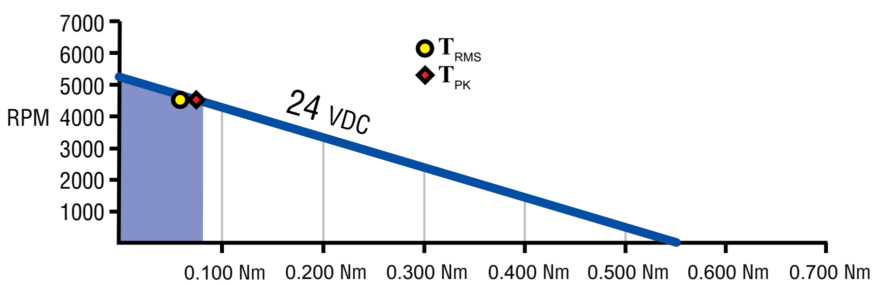
RMS Current Required

$$I_{RMS} = \frac{T_{RMS}}{K_T} + I_O = \frac{0.0640\ \text{Nm}}{0.042\ \text{Nm/A}} + 0.180\ \text{A} = \boxed{1.70\ \text{A}}$$

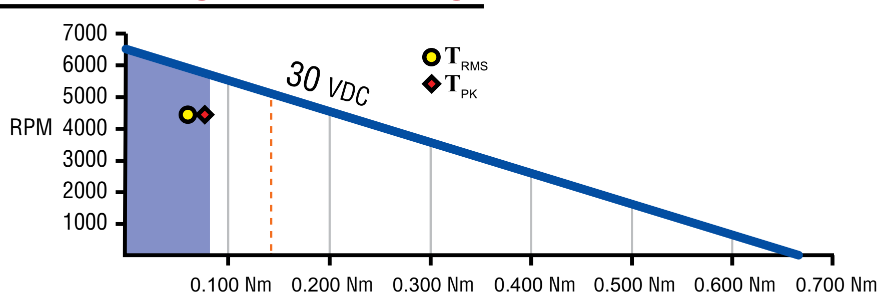
Minimum bus Voltage Required

$$V_{bus} = I_{PK}R_{mt} + \omega_{PK}K_E = (2.10\text{A})(1.85\Omega) + (463.8\ \text{rad/s})(0.042\ \text{V/rad/s})$$
$$= 3.89\ \text{V} + 19.48\ \text{V}$$
$$= \boxed{23.37\ \text{V}}$$

Performance using 24V reference voltage



Performance using a 30V bus voltage



A 30 VDC bus will meet the application requirements as wells supply a margin of safety.

Will the motor meet the temperature rise caused by the application?

Ambient temperature	Θ_a	= 30°C
Rated motor temperature ...	Θ_{rated}	= 155°C
Temperature rise	Θ_r	= 76.38°C
Motor temperature	Θ_m	= 106.38°C

$$\Theta_r = \frac{R_{th} \times I_{RMS}^2 \times R_{mt}}{1 - (R_{th} \times I_{RMS}^2 \times R_{mt} \times 0.00392/°C)}$$

$$= \frac{11°C/\omega \times 1.70A^2 \times 1.85\Omega}{1 - (11°C/\omega \times 1.70A^2 \times 1.85\Omega \times 0.00392/°C)}$$

$$= \frac{58.81}{1 - (0.23)} = \frac{58.81}{0.77} = \boxed{76.38°C}$$

$$\Theta_m = \Theta_r + \Theta_a = 76.38°C + 30°C = \boxed{106.38°C}$$

Haydon kerk Motion Solutions
203 756 7441
1500 Meriden Road
Waterbury, CT USA 06705

PITTMAN
267 933 2105
534 Godshall Drive
Harleysville, PA USA 19438

AMETEK ADVANCED MOTION SOLUTIONS

www.HaydonKerkPittman.com